

Holographic Principle and the Surface of Last Scatter

Paul Howard Frampton ^{a,b*}

^a *Institute for the Physics and Mathematics of the Universe,
University of Tokyo, Kashiwa, Chiba 277-8568, Japan and*

^b *Department of Physics and Astronomy,
University of North Carolina, Chapel Hill, NC 27599-3255, USA.*

Abstract

Using data, provided by WMAP7, I calculate the entropy of the visible universe, where visible refers to electromagnetic radiation, and hence the visible universe is bounded by the Surface of Last Scatter. The dimensionless entropy, S/k , is (8.85 ± 0.37) times larger than allowed by a simplified and non-covariant version of the holographic principle, that the entropy cannot exceed that of a black hole. The measurement of a shift parameter, introduced by Bond, Efstathiou and Tegmark in 1997, plays an important role in the accuracy of the calculation, which leads to the large discrepancy.

*Electronic address: paul.frampton@ipmu.jp; Electronic address: frampton@physics.unc.edu

I. INTRODUCTION

An interesting and profound idea about the degrees of freedom describing gravity, is the holographic principle [1, 2].

For the case of a sphere, with mass M , of radius R , where R will be the co-moving radius for the expanding universe, a simplified, and non-covariant form, of the holographic principle, states that the entropy, S/k , has an upper limit equal to that of a black hole, *i.e.*

$$\left(\frac{S}{k}\right) \leq \left(\frac{S}{k}\right)_{BH} = \left(\frac{4\pi R_S^2}{l_{Planck}^2}\right) \quad (1)$$

where G is Newton's constant, $R_S = 2GM$ is the Schwarzschild radius and l_{Planck} is the Planck length. It should be emphasized that Eq.(1), to which counterexamples are rife, is not the same as the generally covariant holographic principle, enunciated in terms of null hypersurfaces, in [3], as a generalization of Eq. (1). Nevertheless, it is interesting, from the viewpoint of the physical understanding of the visible universe, to use accurate observational data to check, whether the simplified, and non-covariant, Eq.(1) is satisfied at the present time, $t = t_0$, and in the past, cognizant that, with dark energy, if R sufficiently increases, Eq.(1) might, in any case, eventually be violated. Note that, at the time of [1], before dark energy, if Eq.(1) is now satisfied, one might expect it to remain so.

The inequality, Eq.(1), is believed to be saturated by a black hole, although there is no experimental evidence, for such a statement.

The holographic principle is supported, by string theory. The AdS/CFT correspondence [4] is an explicit realization of Eq.(1), and so, apart from the non-trivial subtlety that our universe is dS, not AdS, from the viewpoint of string theory, there is every reason to believe the covariant holographic principle, and to wish to check Eq.(1). It is related to recent considerations of the entropy of the universe [5–7].

However, physics is an empirical science, and therefore the scientific method dictates that we should find a physical example, in which Eq.(1) can be calculated. The result, reported here, is that a detailed and accurate check of Eq.(1), as applied to the visible universe, fails,

by a statistically-significant amount, although in the past, a few billion years ago, it was satisfied.

I should define, precisely, what is meant by the visible universe. It is the sphere, centered for convenience at the Earth, and with a radius $d_A(Z^*) = 14.0 \pm 0.1 Gpc$. The value of $d_A(Z^*)$ is the particle horizon corresponding to the recombination red shift $Z^* = 1090 \pm 1$, and is measured directly by WMAP7 [8], without needing the details of the expansion history. Thus, "visible" means with respect to electromagnetic radiation.

II. THE VISIBLE UNIVERSE

The notion is that the visible universe, so defined, is a physical object which should be subject to the holographic principle. It is an expanding, rather than a static, object, yet my understanding is that the principle, at least in its covariant form, is still expected to be valid.

I shall use the notation employed by the WMAP7 paper [8], from which all observational data are taken.

The present age, t_0 , of the universe is measured to be

$$t_0 = 13.75 \pm 0.13 Gy \tag{2}$$

The comoving radius, $d_A(Z^*)$, of the visible universe, is, likewise, measured to one percent accuracy, as

$$d_A(Z^*) \equiv (1 + Z^*)D_A(Z^*) = c \int_{t^*}^{t_0} \frac{dt}{a(t)} = 14.0 \pm 0.1 Gpc \tag{3}$$

where it is noted that the measurement, of $d_A(Z^*)$, does not require knowledge, of the expansion history, $a(t)$, for $t^* \leq t \leq t_0$.

The critical density, ρ_c , is provided by the formula

$$\rho_c = \left(\frac{3H_0^2}{8\pi G} \right) \quad (4)$$

whose value depends on H_0 , as does the total, baryonic plus dark, matter density, ρ_m

$$\rho_m \equiv \Omega_m \rho_c \quad (5)$$

Because the error on the Hubble parameter, H_0 , is several per cent, it is best to avoid H_0 , in checking the holographic principle.

The mass of the matter, $M(Z^*)$, contained in the visible universe, is

$$M(Z^*) = \frac{4\pi}{3} d_A(Z^*)^3 \rho_m \quad (6)$$

and the Schwarzschild radius, $R_S(Z^*)$, is given by

$$R_S(Z^*) \equiv 2GM(Z^*) \quad (7)$$

Collecting results enables the desired accurate check of the simplified holographic principle, which compares entropy, S/k , for the visible universe, $(S/k)_{V.U.}$, with entropy, $(S/k)_{B.H.}$, for a black hole, of the same mass. According to Eq.(1), this requires

$$\left[\frac{\left(\frac{S}{k} \right)_{V.U.}}{\left(\frac{S}{k} \right)_{B.H.}} \right] \leq 1 \quad (8)$$

A shift parameter, R , was defined by Bond, Efstathiou and Tegmark (BET) in [9], as

$$R = \frac{\sqrt{\Omega_m H_0^2}}{c} (1 + Z^*) D_A(Z^*) \quad (9)$$

which was, with great prescience, introduced by BET, as a dimensionless quantity, to be measured, accurately, by CMB observations.

This BET shift parameter, R , of Eq. (9), is given in [8], as

$$R = 1.725 \pm 0.018 \quad (10)$$

A little algebra shows that the BET shift parameter R provides the most accurate available check, of the holographic principle, by virtue of the result

$$\left[\frac{\left(\frac{S}{k}\right)_{V.U.}}{\left(\frac{S}{k}\right)_{BH}} \right] \equiv R^4 = 8.85 \pm 0.37 \quad (11)$$

showing a violation, by 21σ , of Eq.(8).

III. DISCUSSION

To my knowledge, the visible universe is, at present, the only physical object, for which it is possible to calculate, and compare with experiment, or observation, the simplified holographic principle.

From Eq. (8), the radius $d_A(Z_{HP}) = (1 + Z_{HP})D_A(Z_{HP})$, at which the violation of Eq.(1), begins, is $d_A(Z_{HP}) = 8.4 \pm 0.1 Gpc$, at a time, comparable to when the cosmic deceleration ends, and becomes acceleration. This is strongly supportive of the idea of an entropic accelerating universe, as discussed in [6].

The original aim, of the present work, was to confirm, at $t = t_0$, the inequality, Eq.(8). It was, therefore, surprising to learn that it is violated, with high statistical significance, and has been so, for billions of years.

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